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Dynamical origin of decoherence in classically chaotic systems

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Abstract

The decay of the overlap between a wave packet evolved with a Hamiltonian $\mathscr{H}$ and the same state evolved with $\mathscr{H}+\Sigma$ serves as a measure of the decoherence time τ_ϕ. Recent experimental and analytical evidence on classically chaotic systems suggest that, under certain conditions, τ_ϕ depends on $\mathscr{H}$ but not on Σ. By solving numerically a Hamiltonian model we find evidence of that property provided that the system shows a Wigner–Dyson spectrum (which defines quantum chaos) and the perturbation exceeds a crytical value defined by the parametric correlations of the spectra. © 2000 Elsevier Science B.V. All rights reserved.

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The existence of chaos in classical mechanics is manifested in the evolution of a state as an extreme sensitivity to the initial conditions. Quantum mechanics, on the contrary, does not show this sensitivity. This has raised several problems in a dynamical definition of quantum chaos. In particular, numerical [1] and experimental [2] studies show that time reversal can be achieved with great accuracy. Therefore, the search for a quantum definition of chaos, lead to investigate the spectral properties [3] of quantum systems whose classical equivalent is chaotic. Quantum chaos appears as the regime in which the properties of the eigenstates follow the predictions of the random matrix theory (RMT). In particular, the normalized spacing between energy levels $s=(\varepsilon_{i+1}-\varepsilon_i)/\Delta\varepsilon$, with $\Delta\varepsilon$ the mean level spacing, should have a probability distribution given by the Wigner–Dyson distribution $P^O_{WD}(s)=(\pi s/2)\exp(-\pi s^2/4)$ for an orthogonal ensemble and $P^U_{WD}(s)=(32s^2/\pi^2)\exp(-4s^2/\pi)$ for the unitary ensemble.

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An infinite set of interacting spins is an example of a *many-body* system which is chaotic in its classical version (lattice gas) and hence it is expected to present the quantum signatures of chaos in the spectrum. The dynamics of this particular system can be studied by nuclear magnetic resonance (NMR). Surprisingly, the "diffusive" dynamics of a local excitation $\exp[-\mathrm{i}\mathscr{H}t_R]|0>$ can be reversed [4], generating a Polarization Echo M at time $2t_R$. In this case, $\mathscr{H}$ is the many-body Hamiltonian of a network of spins with dipolar interaction. To accomplish this echo, the transformation $\mathscr{H} \to -(\mathscr{H}+\Sigma)$ at time t_R is performed with standard NMR techniques [4]. This transformation is possible due to the anisotropic nature of the dipolar interaction. The perturbation Σ is a non-invertible component of the Hamiltonian. In some systems, the only contribution to Σ is proportional to the inverse of the radio frequency power and hence can be made arbitrarily small. In a general case the polarization echo (i.e., magnitude of the excitation recovered at time $2t_R$) can then be written exactly as

$$M(t)=|\langle 0|\exp[\mathrm{i}(\mathscr{H}+\Sigma)t/\hbar]\exp[-\mathrm{i}\mathscr{H}t/\hbar]|0\rangle|^2\,, \tag{1}$$

where $|0\rangle$ is the initial wave function, $\mathscr{H}$ the unperturbed Hamiltonian and Σ the perturbation which can be associated with an environmental disturbance. Then the magnitude of experimental interest is the overlap between the same initial wave function evolved with the two different Hamiltonians, $\mathscr{H}$ and $(\mathscr{H}+\Sigma)$. We should note that the second evolution can be seen as an "*imperfect time reversal*" of the wave function! Consistently, more than 10 years ago Peres [5] proposed that dynamical signatures of quantum chaos should be searched on the sensitivity to perturbations in the Hamiltonian. Actually, for a classically chaotic system a perturbation in the initial conditions is equivalent to a perturbation in the Hamiltonian. The experiments show that M decays rapidly with t_R with a Gaussian law [6] indicating a progressive failure in rebuilding the original state. We can define a decoherence time τ_ϕ from this failure, as the width of the Gaussian. This is found [7] to be roughly independent of Σ and it extrapolates to a finite value when $\Sigma \to 0$. Using a semiclassical *one-body* analytical approach in classically chaotic systems characterized by a Lyapunov exponent λ, Pastawski and Jalabert [8] have shown that there is a regime where an exponential attenuation of M is independent of the perturbation Σ with $1/\tau_\phi=\lambda$. This non-perturbative result is valid for long times and as long as Σ does not change the Hamiltonian nature. Our general aim is to find numerical evidence of this regime where $M(t)$ is independent of Σ considering the simplest Hamiltonian that could model spin diffusion.

In this work, we study one-body Hamiltonians in the quasi-1D system with N states which we called *the Stars necklace model*. More specifically, we use a tight-binding model of a ring-shaped lattice with on-site disorder, hopping matrix elements V, and a magnetic flux Φ perpendicular to the plane of the ring (see the inset of Fig. 1). Let us discuss the general features through one representative class, each star has 20 sites and there are $L=35$ beads in the necklace which makes $N=700$. In our case, perturbation acts only between two star beads: $\Sigma(\delta\Phi)=V\exp[\mathrm{i}2\pi\Phi/\Phi_0](\exp[\mathrm{i}2\pi\delta\Phi/\Phi_0]-1)|1\rangle\langle L|+$ *c.c.* Bra and ket states contain orbitals in the star. The localized wave packet with energy $\langle 0|\mathscr{H}|0\rangle \simeq 0$ moves along the string and contains only $3N/4$ states. Anderson

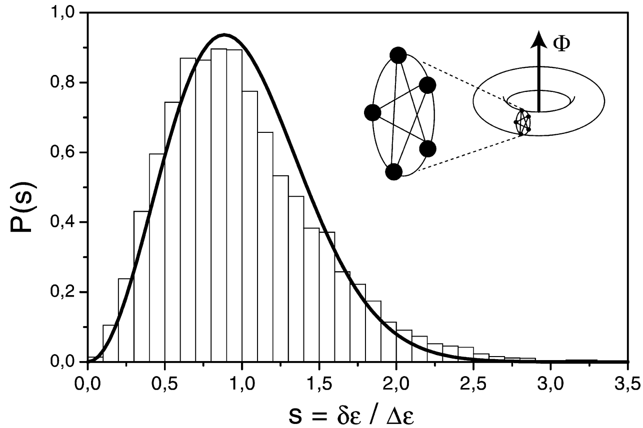

Fig. 1. Probability distribution of the normalized energy level spacing s for the system studied. The black line is the Wigner–Dyson distribution for the unitary ensemble, which is in reasonable agreement with the numerical data. In the inset is shown the schematics of the Hamiltonian model, where the system has the shape of a ring, and the in-layer sites are fully connected.

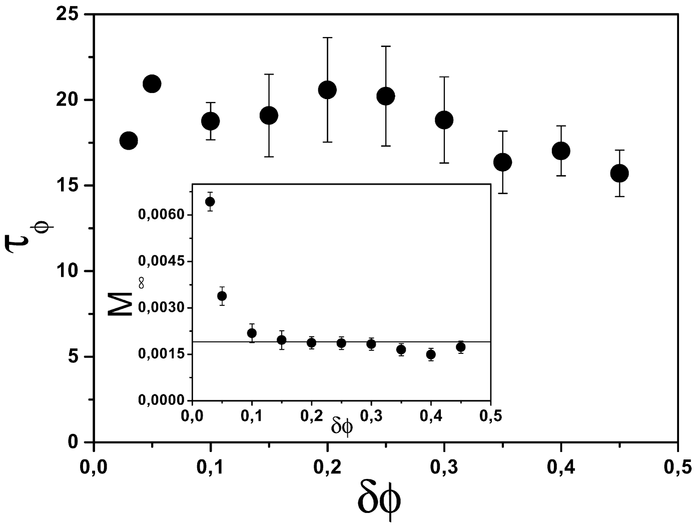

Fig. 2. Plot of the decoherence time of the system according to 2 as a function of $\delta\Phi$. The curve is for a system with 20 sites per layer and a diameter of 35 sites, the amount of disorder is $W = 3$. The inset shows the dependence of the asymptotic value M_∞ with $\delta\Phi$. The straight line corresponds to $4/3N$. As can be seen, M_∞ reaches this value when $\delta\Phi > \delta\Phi_c$.

disorder is $W = 3V$ which gives a $\tau_{\mathrm{imp}} \simeq 3\hbar/V$ and $\Phi = 0.1\Phi_0$. We verify that for $\delta\Phi = 0$ the dynamics of the system follows a diffusive law, and that for all $\delta\Phi$ the statistics of the eigenvalues correspond to those predicted by RMT (see Fig. 1). We interpret these facts as a signature of chaos. However, when studying the parametric correlations of the energy spectrum [9] as a function of $\delta\Phi$ the correlation function is

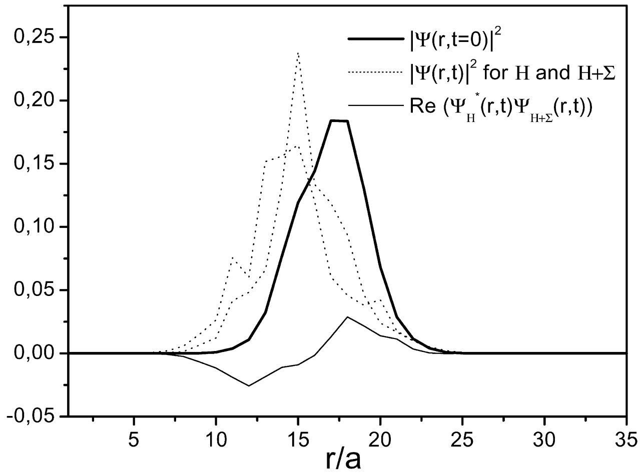

Fig. 3. This figure shows that while the wave functions evolved with $\mathscr{H}$ and $\mathscr{H}+\Sigma$ have very similar probability densities, the overlap between them decays rapidly to zero due to interference effects. In thick solid line, profile of the initial wave function and overlap of the two initial wave functions as a function of the layer, respectively. In thin solid lines, profile of the wave functions evolved with two different Hamiltonians as a function of the layer. Thick dotted line, overlap between the two evolved wave functions. The system has 15 sites per layer and a diameter of 35 sites, $W=3$, $t=2\hbar/V$, and $\delta\Phi=0.1\ \Phi_0$. The sum over layers for this overlap is equal to 1.4 10^{-2}.

definite positive. A critical value $\delta\Phi_c \sim 0.1\ \Phi_0$ can be extrapolated from the strong decay.

For small t the decay of $M(t)$ is Gaussian like with a characteristic time scaling with Σ exactly as could be predicted by a perturbation theory [5]. After a certain time of the order of the collision time τ_{imp} it becomes a stretched exponential with a characteristic time τ_ϕ independent of Σ,

$$M(t) \sim \exp(-t/\tau_\phi)^\nu + M_\infty \tag{2}$$

with $\nu \sim 0.87$ and $\tau_\phi \sim 18\hbar/V$ (see Fig. 2). The constant M_∞ arises from finite size effects. Nonetheless, if the perturbation does not exceed the critical threshold consistent with that of the correlation function, M_∞ increases with decreasing Σ. On the other hand, if the perturbation is large $\delta\Phi > \delta\Phi_c$, one gets $M_\infty \sim 1/N$ (see the inset of Fig. 2). The fact that Σ_c goes to zero when N goes to infinity [9], together with the results of Fig. 2 for a finite system, could be a signature of the existence of a nontrivial thermodynamic limit $\lim_{\Sigma\to 0}\lim_{N\to\infty} M_\infty = 0$ different from the non-thermodynamic one $\lim_{N\to\infty}\lim_{\Sigma\to 0} M_\infty = 1$. Preliminary results on the variation of τ_ϕ with the amount of disorder and system size are consistent with this hypothesis.

In order to present in a graphical way the physical phenomena of decoherence, we calculated the weight of the wave functions that evolved with the two different Hamiltonians and the overlap between them as a function of the layer index. The results are shown in Fig. 3. It can be seen that the evolution of the probabilities described by the perturbed and unperturbed Hamiltonians are not significantly different, i.e., they would give the same coarse-grained values. However, the perturbation causes spatial fluctuations in the phases which produce an integral overlap that decays to zero.

To sum up, our numerical calculations of the polarization echo M in a simple chaotic model indicate that there is a regime of the perturbation where the decoherence time does not depend on the perturbation. This basic feature is also found in experiments [7] and in other theoretical models. Some differences in the details remain, such as the value of the exponent v. According to preliminary numerical evidence v could be related to the particular topology [10] induced by the matrix elements in the Hamiltonian model. Different systems such as disordered cylinders, maximally connected Hamiltonians (RMT) and chaotic stadiums should be studied in order to characterize the details of the dependence of τ_ϕ on $\mathscr{H}$.

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